Assignment 3

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3a)

T(n) = 1 when n = 1

T(n) = 2\*T(n-1) + n + 1 when n > 1

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| T(1) = 1 |  | 1 |
| T(2) = 2\*T(1) + 2 +1 | T(2) = 2 +3 | 5 |
| T(3) = 2\*T(2) + 3 +1 | T(3) = 2(5) + 4 | 14 |
| T(4) = 2\*T(3) + 4 +1 | T(4) = 2 (14) + 5 | 33 |
| T(5) = 2\*T(4) + 5 +1 | T(5) = 2(33) + 6 | 72 |
| T(6) = 2\*T(5) + 6 +1 | T(6) = 2(72) + 7 | 151 |
| T(7) = 2\*T(6) + 7 +1 | T(7) = 2(151) + 8 | 310 |
| T(8) = 2\*T(7) + 8 +1 | T(8) = 2(310) + 9 | 629 |

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| n | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| T(n) | 1 | 5 | 14 | 33 | 72 | 151 | 310 | 629 |

|  |  |
| --- | --- |
| 1) | T(n) = 2\*T(n-1) + n + 1  T(n-1) = 2\*T((n-1)-1)) + (n-1) + 1  = 2\*T(n-1) + n |
| 2) | T(n) = 2 \* [ 2\*T(n-2) + n] + n + 1 = T(n-2) + 2n + n + 1  = 4 T(n-2) + 3n + 1  T(n-2) = 2 \* T ((n-2)-1) + (n-2) +1  = 2 T (n-3) + n - 1 |
| 3) | T(n) = =  =    T(n-3) = 2 T((n-3)-1) + (n-3) +1 = 2 T(n-4) + n - 2 |
| 4) | T(n) = =  =  T(n-4) = 2 \*T((n-4)-1) + (n-4) + 1 = 2\*T(n-5) + n - 3 |
| 5) | = |
| k) | T(k) = +  T(n-k) = T(1) 🡪 n – k = 1  T(n) =  =  =  =  = 3  T(n) = |

3b) Prove by induction:

Claim: for any n , T(n) = 5\*

Proof:

Base Case: n = 1, T(1) = 1

*Explicit Formula:*

Induction Step:

Assume: for any k

P(k):

Then:

T(k+1) =

=

n-k = 1 🡪 k = n-1 🡪 n = k+1

T(k+1) = 2T(k) + (k+1) + 1

= 2() + k + 2

= 10(

= 10(

= )

= 5)

With the induction proof, it is shown that P(k) allows for P(k+1) to be true for all values of n 1. Thus the claim that

T(n) = 5\* is true for all values of n 1.

Q4. Prove that log(n!) is (n log n)

1. Showing log(n!) is O(n log n) where n n0 and

log(n!) c n log n

log(n!) = log(n) + log(n-2) + log(n-3) + … + log n

log(n) + log(n) + log(n) + … + log n

= n log n

if c = 1, n0 = 1: log(n!) c n log n

thus log(n!) is O(n log n)

1. Shows that log(n!) is (n log n)

log(n!) = log n + log(n-1) + log(n-2)+ … + log( + 2) +

log( + 1) + log( )

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if n 4 🡪

=

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it is shown that log(n!) n log n

thus log(n!) is

Because log(n!) is O (n log n) and log(n!) is (n log n),

log(n!) both grows slower than or equal to (n log n) and grows greater than or equal to (n log n). For this to be true, log(n!) must be (n log n).